Internationality: Women in Felix Klein's Courses at the University of Göttingen (1893–1920)

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Renowned for his achievements in mathematics and its applications, Felix Klein (1849–1925) was also instrumental in spearheading the reform of mathematical education. From the early stages of his career, he was internationally oriented and supported mathematically gifted students regardless of their sex, religion, and nationality. The focus of this paper is Klein's role as one of the foremost promoters of women studying mathematics. In these efforts, of course, he was not alone. Klein cooperated with a number of international colleagues who likewise supported women mathematicians, including Gaston Darboux (1842–1917) in France,[1] Luigi Cremona (1830–1903) in Italy, Arthur Cayley (1821–1895) in the United Kingdom, Hieronymus G. Zeuthen (1839–1920) in Denmark,[2] and James Joseph Sylvester (1814–1897)[3]. Since the 1890s, when he founded the acclaimed international center of mathematics at the University of Göttingen, Klein admitted not only male mathematicians from abroad into his courses but non-German women as well.

The goal of this contribution is to evaluate previous scholarship on the beginning of women's study of mathematics at German universities and to analyze the special efforts of Felix Klein to advance this cause. It will also be shown when the first female mathematicians joined the German Mathematical Society, which was founded in 1890, and when female authors first published in the journal *Mathematische Annalen*,[4] the chief editor of which was Klein himself. My study is based on materials from Klein's archive in Göttingen, especially on the lists of students enrolled in his courses and on the protocols from his mathematics seminars.

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[1] See Tina Richter (2015), pp. 18–20; and Tobies (2016).

[2] Zeuthen and Johan Ludvig Heiberg (1854–1928) supervised Thyra Eibe (1866–1955), the first women in Denmark to complete her doctorate in mathematics in 1895 (see below).

[3] As early as 1878, Christine Ladd-Franklin (1847–1930) was admitted to study at Johns Hopkins University under the supervision of the British mathematician Sylvester (see Fenster and Parshall [1994], p. 234). Felix Klein was asked to succeed Sylvester in 1883, when the latter returned to Great Britain.

[4] See Tobies & Rowe (1990).

Non-German Women Paving the Way in Germany

Although women were not legally permitted to enroll in German universities during the nineteenth century, the first women to earn a doctoral degree in mathematics there nevertheless did so at the University of Göttingen in 1874: the Russian Sofia Kovalevskaya (1850–1891). The life and work of Kovalevskaya – and the circumstances of her doctorate *in absentia* – have received sufficient scholarly attention.[5] It should be stressed that Kovalevskaya's career, until she became a full professor in Stockholm, had been assisted by mathematicians from Sweden, Germany, France, and Italy.[6] Felix Klein, too, then a young professor at the University of Erlangen, immediately recognized the significance of Kovalevskaya's thesis and praised it to the Norwegian Sophus Lie (1842–1899).[7]

Kovalevskaya's career, however, was an exception, and it was not until 1895 that the next women mathematicians completed their doctorates at German universities.[8] Marie Gernet (1865–1924) was the first German to do so, namely at the University of Heidelberg and under the direction of Leo Königsberger (1837–1921),[9] with whom Kovalevskaya had begun her studies in 1869. In 1895, too, the Englishwoman Grace E. Chisholm (1868–1944)[10] and the American Mary F. Winston (1869–1959) submitted their doctoral dissertations, both of which were written under the direction of Felix Klein in Göttingen. It was in the same year, incidentally, that the aforementioned first female Danish mathematician received her doctoral degree in Copenhagen: Thyra Eibe.[11]

From an international perspective, between the year 1874 (the year of Kovalevskaya's doctorate) and 1895, five other female mathematicians received a doctoral degree: the Russian Elizaveta Fedorovna Litvinova (1845–1919) – a friend of Kovalevskaya's – at the University of Bern in 1878; the Englishwoman Charlotte Angas Scott (1858–1931) in London in 1885; and three North-Americans: Winifred Harington Edgerton (1862–1951), who was the first to do so in the United States, with a thesis submitted at Columbia University in 1886; Ida

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[5] Kovalevskaya's PhD examination records are published in Tollmien (1997); see also Eva Kaufholz-Soldat's contribution in this book. TODO (o^.^o)

[6] On the support that Kovalevskaya received from French and Italian mathematicians, see Coen (2012), pp. 477, 509–515.

[7] Kovalevskaya's thesis was published as "Zur Theorie der partiellen Differentialgleichungen," *Journal für die reine und angewandte Mathematik* 80 (1875), pp. 1–32. Klein's remarks to Lie about the thesis begin as follows: "What do you think about Sophia Kovalevskaya's study in Borchardt's journal? By means of direct series expansion, she proves the existence of integrals as well as their definiteness within certain limits." Quoted from a letter from Klein to Lie dated July 8, 1875.

[8] Regarding the laws governing the enrollment of women in several German states, see the details in Tobies (1997) and Birn (2015).

[9] Marie Gernet became a teacher at the first German secondary school for girls where it was possible to take the *Abitur*, the examinations required for entrance to German universities. On the school, which was founded in Karlsruhe in 1896, see Tobies (2001).

[10] See Elisabeth Mühlhausen's chapter II.1.

[11] See footnote 2. Regarding the development of the Danish educational system, see Lisbeth Fajstrup, Anne Katrine Gjerløff and Tinne Hoff Kjeldsen's chapter III.3.

Martha Metcalf (1857–1952) from Cornell University in 1893; and one year later, likewise from Cornell, the Canadian Annie Louise MacKinnon (1868–1940).[12] MacKinnon went on to conduct post-doctoral research in Göttingen (see the table below), after which continued her career in the United States, where there were better job opportunities.

At the University of Cambridge in United Kingdom, a remarkable number of women graduated in mathematics, first from Girton College in 1873 and then from Newnham College in 1875.[13] But they could neither earn a doctoral degree there nor receive a research position. On these grounds, some of them tried their luck abroad. Recommended by Arthur Cayley, Charlotte Angas Scott became chair of the mathematics department at the newly-founded Bryn Mawr College for women in Pennsylvania (United States), where she maintained contact with Göttingen.[14] Her association with Göttingen was based above all on her achievements in algebraic geometry, which included a proof of a theorem by Max Noether (1844–1921). The latter proof was published in the journal *Mathematische Annalen* in 1899.[15] Scott was actively involved in the American Mathematical Society, for which she served as vice president in 1905 and 1906. Moreover, she became the first female member of the German Mathematical Society (*Deutsche Mathematiker-Vereinigung*) in 1898, when Felix Klein was its president.[16] Scott, who supervised doctoral theses at Bryn Mawr, arranged for some of her students to pursue post-doctoral studies at the University of Göttingen. On March 19, 1897, for example, she wrote the following to Klein: "I expect to send two of my best students to Göttingen next year. Both have been awarded a College Fellowship, and both are eager to study under your direction for a year, if this is agreeable to you."[17] Thus in the fall of 1897, Emilie Norton Martin (1869–1936) and Virginia Ragsdale (1870–1945)[18] arrived in Göttingen along with other American students. Having benefitted from their time with Klein and Hilbert, both went on to complete their doctorates under Scott's supervision at Bryn Mawr. In 1910, as though in exchange, Klein's youngest daughter Elisabeth (1888–1868) spent a semester abroad at Bryn Mawr. It was at that welcoming university, too, where Emmy No-

[12] On MacKinnon (married name: Fitch, as of 1901) and other early North-American women in mathematics, see Fenster & Parshall (1994), p. 235; and Green & LaDuke (2009).

[13] See Davis's archive of female mathematicians, which includes a chronological list of graduates from the University of Cambridge (1873–1940):
http://www-history.mcs.st-and.ac.uk/Davis/Indexes/xCambridge.html

[14] See Grinstein and Campbell (1987); see also Parshall (2015).

[15] Charlotte Angas Scott, "A Proof of Noether's Fundamental Theorem," *Mathematische Annalen* 52 (1899), pp. 593–597.

[16] Toepell (1991), p. 354. It was not until its one hundredth anniversary that the DMV would elect a female mathematician to its council, in 1991: the algebraist Ina Kersten, a scholarly descendent of Emmy Noether. She became president from 1995 to 1997.

[17] [UBG] Cod. Ms. Klein XI, p. 947.

[18] Ragsdale contributed to Hilbert's "sixteenth problem" (Ragsdale conjecture), as did Hilbert's German doctoral students Margarete Kahn and Klara Löbenstein.

ether would find refuge in 1933, and Hilda Geiringer (1893–1973) would do the same from 1939 to 1944.[19]

It should be stressed that David Hilbert (1862–1943), who became a full professor at the University of Göttingen in 1895 with Klein's endorsement, was also a staunch supporter of women's right to study. Together, Klein and Hilbert conducted seminars in which women took part (see the table below). Hilbert would supervise sixty-nine doctoral students in all, six of whom were women: Anne Lucy Bosworth (1868–1907) from the United States; Nadeschda Nikolaevna von Gernet (1877–1943), Ljubov Nikolaevna Zapolskaya (Sapolsky, Sapolski) (1871–1943)[20], and Vera Lebedeva (1880–1970) from Russia; and Margarete Kahn (1880–1942) and Klara Löbenstein (*1883) from Germany.[21]

I should add that it was not in Germany alone that foreigners paved the way for native women interested in science and mathematics. The situation was similar in France.[22] A well-known case is that of Maria Skłodowska–Curie (1867–1934), who earned a *Lizenziat* in physics (1893) and mathematics (1894) at the Sorbonne. However, the aforementioned Sofia Kovalevskaya had already become a member of the French Mathematical Society (*La Société Mathématique de France*) as early as 1882. Kovalevskaya's Finnish student Ebba Louise Nanny Lagerborg (later Cedercreutz; 1866–1950) would also complete her *Lizenziat* at the Sorbonne and become a member of the French Mathematical Society (in 1890).[23]

The First Female Members of the German Mathematical Society

Whereas the French Mathematical Society had been founded in 1874, the German Mathematical Society (*Deutsche Mathematiker-Vereinigung* = DMV) was not formed until in 1890, the year of Kovalevskaya's death. The first women to become members of the DMV were foreigners. The aforementioned Charlotte Angas Scott (1898) was followed by Hilbert's doctoral student Nadeschda von Gernet in 1901. In 1904, the American Helen Abbot Merrill (1864–1949) became a member.[24] She had studied at the University of Göttingen from 1901 to 1902 and earned a doctoral degree from Yale two years later.[25] Elizabeth Buchanan Cowley

[19] On Elisabeth Klein (married name: Staiger), see Tobies (2008). Regarding women mathematicians in Germany who had to emigrate because of the Nazi dictatorship, see Tobies (2011b); and Siegmund-Schultze (2009).

[20] Regarding Zapolskaya's biography, see Makeev (2011). She received a teaching position at the University of Moscow, published her results in books, and became the first Russian woman with a post-doctoral degree and a professorship in 1905. The professional path that led her there was winding, however. She had been the headmaster of a secondary school and a lecturer at institutions in Moscow, Ryazan, Saratov, and Yaroslavl.

[21] See Tobies (1999); and König, Prauss & Tobies (2014).

[22] This is based on a lecture given by Catherine Goldstein at the University of Würzburg in October of 2015.

[23] See https://fi.wikipedia.org/wiki/Nanny_Cedercreutz (accessed October 30, 2016).

[24] Toepell (1991), p. 254.

[25] Singer (2003), p. 93.

(1874–1945) became the next American female member in 1908.[26] In that year, she had received her PhD from Columbia and embarked upon further studies at the Universities of Göttingen and Munich. In 1907, she and Ida Whiteside (*1883) published an article together in the journal *Astronomische Nachrichten*, for which they were awarded a prize from the German Astronomical Society.[27]

The first Italian joined the DMV in 1905: Laura Pisati. She had earned her doctoral degree in Rome in 1903, taught at the Technical School "Marianna Dionigi," and was also – as of February 26, 1905 – a member of the Circolo Matematico di Palermo.[28] Her article "Sulla estensione del metodo di Laplace alle equazioni differenziali lineari di ordine qualunque con due variabili indipendenti" was long a fixture in scholarly bibliographies.[29] On account of her tragic premature death, Pisati was unable to deliver her lecture "Essay on a Synthetic Theory for Complex Variable Functions" at the Fourth International Congress of Mathematicians in Rome, where she would have been the first woman to have spoken at this event.[30] Although Emmy Noether accompanied her father to this conference, she did not give a talk there. Having just finished her doctorate under Paul Gordan (1837–1912) at the University of Erlangen, she would deliver her first lecture at the conference held by the DMV in Salzburg one year later.[31] In that same year, 1909, she became the first German woman to be granted membership to the DMV.

In subsequent years, a number of other female German mathematicians earned a doctoral degree and became members of the DMV.[32] Additional non-German women joined as well. Here I would like to single out the aforementioned Austrian Hilda Geiringer, who came to Berlin after completing her doctorate in 1918 at the University of Vienna under the supervision of Wilhelm Wirtinger (1865–1945). She, too, was in contact with Felix Klein, and she was made a member of the DMV in 1921.[33] Olga Taussky (1906–1995), another Austrian who completed her doctorate with Klein's former student Philipp Furtwängler (1869–1940) in Vienna,

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[26] Toepell (1991), p. 75.

[27] See https://www.agnesscott.edu/lriddle/women/cowley.htm (accessed August 20, 2016).

[28] See Toepell (1991), p. 291; and Jones (2009), p. 91.

[29] Laura Pisati, "Sulla estensione del metodo di Laplace alle equazioni differenziali lineari di ordine qualunque con due variabili indipendenti," *Rendiconti del Circulo Matematico di Palermo* 20 (1905), pp. 344–374. See further Ganzha, Loginov & Tsarev (2006).

[30] A memorial for Pisati was held during Section I of the Congress in Rome; see Curbera (2009), p. 44. – At the first International Congress of Mathematicians (ICM, Zurich, in 1897), four female mathematicians took part: Iginia Massarini (Rome), Vera von Schiff (St. Petersburg), Charlotte Angas Scott (Bryn Mawr), and Charlotte Wedell (Göttingen); see Eminger (2015), p. 70. On Massarini, see Carbone and Talamo (2010). The first woman to give a talk at an ICM was H. P. Hudson, who spoke at Cambridge in 1912 (see Eminger [2015], p. 70).

[31] Emmy Noether, "Zur Invariantentheorie der Formen von n Variablen," *Jahresbericht der Deutschen Mathematiker-Vereinigung* 19 (1910), pp. 147–154.

[32] See Tobies (2006).

[33] While writing a popular science book on mathematics, which makes use of Klein's educational reform ideas and his conceptual coupling of precision and approximation mathematics, Geiringer wrote two letters to Klein, dated November 7, 1921 and December 3, 1921 (see [UBG] Cod. MS Felix Klein 9, pp. 307–308). Klein had read the manuscript and sent comments to her (see Geiringer [1922], pp. 93–95).

became a member in 1930 when she was called upon in Göttingen to edit certain chapters (on number theory) of Hilbert's collected works. When the Nazis came to power, she, too, was forced to emigrate. Finally, the British mathematician Dorothy Wrinch (1894–1976), a biochemist who applied mathematical principles, joined the DMV in 1933.[34]

Felix Klein and the First Women to Study at the University of Göttingen

Beginning with the winter semester of 1893, three years after the foundation of the DMV, Felix Klein made it possible for the first women to enroll at the University of Göttingen. As mentioned above, these women were at first exclusively foreigners. In order to understand this situation, it is necessary to examine it in closer detail.

The Humboldtian university reform had provided a decisive impetus for mathematical research in Germany, one that attracted an increasing number of foreign students during the last third of the nineteenth century. Both women and men wished to study at the center of scholarly activity. As early as Klein's time in Munich (1875–1880), young men came to study with him from Italy, Norway, and elsewhere. With his move to Leipzig in 1880, French and American students came to learn from him, and when, in 1886, he became a professor in Göttingen, students from Russia came to him as well. Between 1886 and 1895, approximately ten Americans earned a doctoral degree under Klein's supervision. The significance of these numbers becomes clear when we learn that, throughout the 1880s and 1890s in Leipzig and Göttingen, more Americans studied mathematics under Klein and his successor at Leipzig (the Norwegian mathematician Sophus Lie) than under any professor of mathematics in the United States. It goes without saying that the subsequent development of mathematics in that country was emphatically influenced by this contact.[35]

Sandra L. Singer has written a detailed book about North American women at German-speaking universities during the period of 1868 to 1915.[36] Before her, in a pioneering study, Margaret Rossiter underscored Felix Klein's special role in promoting American women mathematicians.[37] Singer made good use of Rositter's findings and drew upon additional archival sources.[38] An outline of the achievements of the first women in the American mathematical community can be found in the work of Della D. Fenster and Karen H. Parshall.[39] David Rowe has underscored the role of Felix Klein's wife Anna (a granddaughter of the famous philosopher Friedrich Wilhelm Hegel), who integrated the first foreign students

[34] See Toepell (1991), pp. 120, 381, 424; and Senechal (2013).
[35] See Parshall & Rowe (1994).
[36] Singer (2003).
[37] Rossiter (1982), pp. 40–41.
[38] Singer (2003), pp. 86–97.
[39] Fenster & Parshall (1994).

into her family and helped them to find their bearings in Göttingen.[40] In my own work, which is likewise based on archival material, I have been able to document the earliest stages of women studying mathematics in Göttingen.[41] There I have shown, for instance, how Klein managed to overcome the conservative attitude of the legal scholar and university curator Ernst von Meier (1832–1911), who in 1891 had prevented Christine Ladd-Franklin and Ruth Gentry (1862–1917) from participating in Klein's courses.[42] The massive amount of resistance faced by Klein and the perseverance required of him to change anything are indicated in a letter to him from Meier, which reads: "This is worse than social democracy, which only wants to do away with differences in property. They want to abolish the difference between the sexes."[43]

While women in other countries were already able to take qualifying examinations and even to study at university, the ministerial decrees allowing women to matriculate in German states were first issued between 1900 and 1909.[44] However, because foreign women, like men, wanted to be qualified to study where the highest standards of scholarship could be expected, for some time they attempted to gain access to German universities even while official status as students could not yet be granted to them.

Up until the beginning of the 1890s, Berlin was regarded as the center of mathematics in Germany, with professors there such as Karl Weierstraß (1815–1897), Ernst Eduard Kummer (1810–1893), and Leopold Kronecker (1823–1891). Around that time in Göttingen, Felix Klein developed an international center for mathematics, which was further enhanced by Hilbert's arrival in 1895. In order to accomplish his goals, Klein sidestepped the prescribed order of command (evading the conservative university curator) and communicated directly with the influential official Friedrich Althoff (1839–1908) at the Prussian Ministry of Culture. On May 20, 1892, the farsighted Althoff issued a new brief with the title "The Request of Persons of the Female Gender to Matriculate and Attend Lectures at the Royal State Universities." This document begins with excerpts from newspapers about the ability of women to study in foreign countries.[45] Developments abroad, in other words, influenced the decisions made at the Prussian Ministry of Culture.

Heinrich Maschke (1853–1908), who had studied under Klein in Göttingen and was a professor of mathematics at the newly established University of Chicago, wrote the following to his former teacher on April 8, 1893:

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[40] See Rowe (1992), chapter 5.7; and Parshall & Rowe (1994), pp. 239–253. For an interpretation of the roles played by the wives of mathematicians, see also Jones (2009), p. 37.

[41] Tobies (1991/1992).

[42] [UBG] Cod. MS F. Klein 9, pp. 310–311 (Gentry's letters to Klein, dated July 21 and 31, 1891).

[43] The original German reads as follows: "Das ist schlimmer als die Sozialdemokratie, die nur den Unterschied des Besitzes abschaffen will. Sie wollen den Unterschied der Geschlechter abschaffen" ([UBG] Cod. MS F. Klein, 22L, p. 7; also quoted in Siegmund-Schultze [1997], p. 31; and in English in Rowe [1992]).

[44] See Costas (2002); and Tobies (1997).

[45] See Tobies (1991/1992), p. 151.

> One of our students of mathematics, Miss Mary F. Winston, is applying for a scholarship, on the basis of which she intends to go to Germany next year. She has [...] talent, thinks independently, and is certainly above average. [...] Bolza[46] and I have encouraged her [...] to go to Göttingen and have just as forcefully discouraged her from going to Berlin in order to keep her away from the stiff atmosphere there. Now the question remains whether female doctoral or post-doctoral students may enroll at Göttingen or whether, if that is not the case, you think you might exert your influence to make an exception.

On July 6, 1893, before he would first travel to the United States (for the World's Fair and a mathematics conference), Klein received the following disheartening message from Berlin:

> With respect to women studying, as I know from Mr. Althoff, it is sad to say that the matter is now such that if questions of this sort are not brought up, no one here will raise any objections. Regarding their participation in lectures, moreover, this custom will become rather more entrenched than limited if American women were to come here for the purpose of studying, all the more so if they are made to face fewer difficulties. Mr. Althoff is of the opinion that, without asking, you would like to have all of your American admirers come over here.[47]

Klein's attitude was reinforced in the United States, where he not only confirmed Mary F. Winston's outstanding talent as a mathematician but also witnessed women in positions that a Swiss delegate to the World's Fair described as follows:

> The Americans find nothing unusual in the fact that, for instance, a woman is the director of a national bank, as in Texas, or that women have found a place on the supervisory committees of universities or in the national department of education, and this is not to mention professorial positions, of which there are many for women [...]. Not only have universities been made available to women but also preparatory secondary education, be it in connection with schools for boys or in parallel institutions, as in Boston [...]. America knows no difference in the practice of scientific careers between men and women [...]. At the University of Chicago, there are six female professors.[48]

Having returned from Chicago to Germany, Klein proposed to the Ministry of Culture in Berlin that Winston, Grace Chisholm,[49] and the American Margaret Eliza Maltby (1860–1944) be allowed to enroll in university. Despite another negative vote by the university curator, the ministry approved the application of these women within six days. The curator resigned from his position in a huff, and his successor was welcoming to women. All three women participated officially in

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[46] Oskar Bolza (1857–1942), who had earned his doctorate in Göttingen in 1886, also became a professor at the University of Chicago. On the developments in the United States, see especially Parshall & Rowe (1994).

[47] Tobies (1991/1992), p. 154.

[48] Boos-Jegher (1894), pp. 8, 16.

[49] Grace Chisholm had studied at Girton College at the University of Cambridge, where women could not earn a doctoral degree. One of her professors of mathematics, Andrew Forsyth (1865–1942), recommended that she go to Felix Klein, with whom he was well acquainted (see Mühlhausen [1995], p. 197).

lectures and seminars and earned doctoral degrees with distinction, Maltby with a dissertation in the field physical chemistry directed by Walther Nernst (1864–1941). The female students were not officially matriculated but rather possessed the status of auditors (every professor had to be asked individually for permission, which ultimately had to be granted on an individual basis by the ministry).

In the meantime, additional female students had arrived at Göttingen. Klein assisted them personally to receive permission to attend courses at the university. In autumn of 1894, he wrote the following to the Prussian minister of education Robert Bosse (1832–1901):

> Your Excellence!
>
> In addition to the two women, Miss Chrisholm and Miss Winstaon, who for the last year have been studying mathematical subjects at the local university and whose diligence and abilities I have repeatedly commended, there are now two new applicants, Miss MacKinnon and Miss Maddison,[50] who are likewise requesting permission from the appropriate instructors to participate, as of the next semester, in lectures on mathematics, physics, and astronomy. I have examined the qualifications of both women and am thus able to support their applications in every respect.[51]

On November 1, 1895, the mathematician Arthur Schönflies, an associate professor of descriptive geometry at Göttingen, wrote the following remarks: "We now have nine women studying mathematics, and yesterday they formed a club; they will meet once a week for coffee."[52] These meetings can be interpreted as the formation of the first women's network of mathematicians. In the same year, because of a growing demand for access, the Prussian ministry determined that universities only to provide it with a list of the female participants in their the courses (as auditors).

When, in 1896–97, Klein was asked about his position concerning women studying and pursuing scientific careers, he offered the following response:

> I am far more pleased to answer this question than that concerning the opinion still prevailing in Germany, which is that the study of mathematics must be as good as inaccessible to women, that there should be an essential blockade to any efforts directed toward the development of women's higher education. In this regard, I am not referring to extraordinary cases, which as such would not prove very much, but rather to our average experiences in Göttingen. Though this is not the place to get into the matter, I would simply like to point out that in this semester, for instance, no fewer than six women have participated in our higher mathematics courses and practica and, having advanced through them, have proven themselves to be equal to their male classmates in every respect. The nature of the situation is that, for the time being, these women have been ex-

[50] Ada Isabel Maddison (1869–1950) was a British woman who, like Scott and Chisholm, studied with Arthur Cayley in Cambridge. After graduating from Girton College in 1892, she attended Bryn Mawr College in the United States, where she won a fellowship for studying abroad. After returning to Bryn Mawr, Maddison completed her PhD and translated Klein's article "The Arithmetizing of Mathematics," *Bulletin of the American Mathematical Society* 2 (1896), pp. 241–249.

[51] [UBG] Cod. Ms. Klein I C2, pp. 95–96.

[52] Quoted from Tobies (1991/1992), p. 157.

> clusively foreigners: two Americans, an Englishwoman, and three Russians. No one would wish to assert, however, that these foreign nations possess some inherent and specific talent that evades us, and thus that, with suitable preparation, our German women should not be able to accomplish the same thing.[53]

Klein's conclusion was that the infrastructure for educating girls should be enhanced in Germany.

Female Participants in Klein's Courses

Klein kept an account of the students who attended his courses. This record extends from his time as a university lecturer (the summer semester of 1871) until the year 1920, even though he had already become a professor emeritus in 1913.[54] The first women appear on his list for the winter semester of 1893/94: Grace E. Chisholm and Mary F. Winston. They attended Klein's lecture on hypergeometric series and also, though only in their first semester, gave presentations in his seminar. His first women students from the German-speaking region were Frieda Hansmann (*1873 in Northeim, Prussia) in the summer semester of 1895 – who would later earn a doctoral degree in chemistry from the University of Bern – and Elsa Neumann (1872–1902), who attended Klein's lecture on technical mechanics in the summer semester of 1896 and would become, in 1899, the first woman ever to be awarded a doctoral degree from the University of Berlin (her field was physics).[55]

The presence of women in university classrooms became an increasingly normal occurrence in Göttingen, although it was not until 1908 that they could attend as more than mere auditors. Together with new laws enacted in Prussia in 1908, which allowed the enrollment of women, and the establishment of new kinds of girls' secondary schools, which could culminate in the *Abitur*, this led to even more women entering higher education. It is also apparent that an increasing number of German women began to enroll in Klein's courses (see the table below). At that time, the preferred career goal of both female and male students of mathematics was to become a secondary school teacher (of mathematics and other subjects).[56]

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[53] Quoted from Kirchhoff (1897), p. 241.
[54] [UBG] Cod. MS F. Klein VII E.
[55] See Vogt (1999).
[56] See Abele, Neunzert & Tobies (2004); and Tobies (2011a).

Semester	*Lectures and Seminars* (Hours per Week)	*Number of Participants* (Male and Female)	*Female Students*
WS 1893/94	*Hypergeometric functions* (4)	11	Mary F. Winston, USA Grace E. Chisholm, UK
	Math. seminar (2): *Linear differential equations and p-functions*	11	M. F. Winston* G. E. Chisholm*[57]
SS 1894	*Linear differential equations of second order* (4)	10	M. F. Winston G. E. Chisholm
	Selected problems of elementary geometry (2)	14	M. F. Winston G. E. Chisholm
	Math. seminar (2): *Linear differential equations and spherical functions*	12	M. F. Winston* G. E. Chisholm*
WS 1894/95	*Number theory* (4)	12	G. E. Chisholm Ada M. Johnson (b. 1870), UK[58] Ada Isabel Maddison, UK, later USA Annie Louise MacKinnon, USA
	Math. Seminar (2), concluded by Heinrich Burkhardt (1861–1914), with the participation of Arnold Sommerfeld (1868–1951): *Foundations of functions with one variable*	19	M. F. Winston* A. I. Maddison* A. L. MacKinnon*
SS 1895	*Differential calculus* (4)	24	Lillien Jane Martin (1851–1943), USA[59] Frieda Hansmann Helene v. Bortkewitsch (1870–1939)[60] Alexandrine v. Stebnitzky (b. 1868 Tbilisi), Russia[61]

[57] See the *Protokolle* of Felix Klein's seminars. The titles of the presentations given by women in Klein's seminars are published in Appendix 7 of Tobies (1991/1992), pp. 166–169. Those who gave one or two presentations in his seminars are indicated here by one or two asterisks (*) (**).

[58] Ada Maria Jane Elizabeth Johnson were educated in Newnham College at University of Cambridge; see http://www-history.mcs.st-and.ac.uk/Davis/Indexes/xCambridge.html

[59] From 1894 to 1898, L. J. Martin studied a variety of subjects in Göttingen, especially psychology.

[60] See Vogt (2014), p. 24.

[61] A. von Stebnitzky was the daughter of the Polish general I. I. von Stebnitzky (1832–1897), a geodesist and a corresponding member of the Academy of Sciences in St. Petersburg. She be-

	Exercises (1)	14	Lillien Jane Martin Frieda Hansmann
	Math. seminar (2), conducted with Hilbert and Ernst Ritter (1867–1895): *Differential calculus*	17	M. F. Winston* H. v. Bortkewitsch* A. v. Stebnitzky* Ada M. Johnson* A. L. MacKinnon* A. I. Maddison*
WS 1895/96	*Number theory* (2)	14	Ljubov N. Zapolskaya H. v. Bortkewitsch A. v. Stebnitzky A. L. MacKinnon Ada M. Johnson
	Theory of the top (2)	19	L. N. Zapolskaya, H. v. Bortkewitsch A. v. Stebnitzky A. L. MacKinnon Mary F. Winston
	Math. seminar (2), conducted with Hilbert and Arnold Sommer-feld: *Number theory*	9	A. L. MacKinnon** A. v. Stebnitzky* Ada M. Johnson* H. v. Bortkewitsch* L. N. Zapolskaya*
SS 1896	*Technical mechanics* (2)	13	Elsa Neumann
	Number theory (2)	9	L. N. Zapolskaya A. L. MacKinnon Ada M. Johnson
	Math. seminar (2): *Number theory*	8	L. N. Zapolskaya* Ada M. Johnson* A. L. MacKinnon*
WS 1896/97	*Integral calculus* (4)	26	L. N. Zapolskaya H. v. Bortkewitsch Charlotte Wedell (b. 17.I.1869)[62], Denmark
	Math seminar (2), Klein and Hilbert: *Theory of functions and conformal mapping*	26	Charlotte Wedell* L. N. Zapolskaya, H. v. Bortkewitsch
SS 1897	*Differential equations* (4)	27	
	Math. seminar (2), Klein and Hilbert: *Theory of functions*	19	L. N. Zapolskaya

came an astronomer in Pulkovo near St. Petersburg and later at the Fort Skala Observatory in Krakow. The famous physicist Piotr L. Kapitza (1894–1984) was her nephew. I am indebted to Danuta Ciesielska for this information.

[62] Wedell, who had studied with Klein's former doctoral student Adolf Hurwitz (1859–1919) at the ETH Zurich, completed her doctorate at the University of Lausanne in Switzerland. The title of her thesis is "Applications de la théorie des fonctions elliptiques à la solution du probléme de Malfatti" (1897). Many thanks to Nicola Oswald for this reference.

WS 1897/98	*Mechanics I* (4)	49	Emilie Norton-Martin, USA Virginia Ragsdale, USA Fanny Cook Gates (1872–1931),[63] USA Katharina Hogdon (b. 1871), USA L. N. Zapolskaya Grace E. Chisholm Young
	Exercises in mechanics (1)	31	
	Math. seminar (2), Klein and Hilbert: *Mechanics*	12	Grace E. Chisholm Young*
SS 1898	*Mechanics II* (4)	37	Emilie Norton-Martin Virginia Ragsdale Fanny Cook Gates Katharina Hogdon L. N. Zapolskaya Grace E. Chisholm Young A. L. Bosworth
	Exercises in mechanics (1)	10	Emilie Norton-Martin V. Ragsdale Fanny Cook Gates Katharina Hogdon L. N. Zapolskaya
	Math. seminar (2), Klein and Hilbert: *Mechanics*	18	
WS 1898/99	*Theory of functions* (4)	54	
	Math. seminar (2): *Analysis of real functions*	14	A. L. Bosworth* Anna Helene Palmié* (married name: Therriel) (1863–1946), USA
SS 1899	*Theory of functions and theory of potential* (4)	21	
	Math. seminar (2): *Theory of functions* (2)	23	Nadeschda Nikolaevna von Gernet*
WS 1899/1900	*Mechanics, especially hydrodynamics* (3)	36	
	Math. seminar (2): *Ship movement*	28	
SS 1900	*Analytic geometry* (4)	42	
	Math. seminar (2) conducted with Max Abraham (1875–1922): *Technical applications of elasticity theory*	12	
WS 1900/01	*Projective geometry* (4)	88	Mary Esther Trueblood (married name: Paine)

[63] Gates became a pioneering woman in the field of radioactivity.

			(1872–1939), USA
	Exercises (1)	29	
	Math. seminar (2): *Applications of projective geometry*	16	
SS 1901	*Applications of calculus on geometry* (4)	77	Mary Esther Trueblood
	Math. seminar (2): *Geodesy*	29	
WS 1901/02	*Mechanics I* (4)	78	
	Math. seminar: *Selected chapters of mechanics*	30	
SS 1902	*Mechanics II* (4)	53	
	Math. seminar (2), conducted with Karl Schwarzschild (1873–1916): *Astronomy*	26	
WS 1902/03	*Encyclopedia of mathematics I* (4)	44	Miss[64] Wassielieff Tatyana Afanasyeva (-Ehrenfest) (1876–1964)[65], Russia Elizabeth Stephansen (1872–1961)[66], Norway
	Math. seminar (2), conducted with Karl Schwarzschild: *Principles of mechanics*	27	T. Afanasyeva*
SS 1903	*Encyclopedia of mathematics II, geometry* (4)	61	T. Afanasyeva, V. Lebedeva, Russia A. H. Palmié
	Math. seminar (2): *Graphical statics and the strength of materials*	24	

[64] All female students were listed as "Frl." (Miss) with the exception of Joukowsky, who appears as "Frau" (Mrs.). Whenever first names could be identified, I have used those instead of "Miss" in the table.

[65] She married the Austrian physicist Paul Ehrenfest (1880–1933) on February 21, 1904. Ehrenfest studied in Göttingen as well. Together, they wrote the famous contribution on statistical mechanics for the third volume of the great *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, which was edited by Klein.

[66] Stephansen was the first women in Norway to receive a PhD in mathematics. From 1892 to 1896, she studied at the ETH Zurich, where she earned a diploma. Later, in 1902, Heinrich Burkhardt (1861–1914), who did his post-doctoral degree under Klein in Göttingen, arranged for Stephansen to be granted her doctorate *in absentia* (the title of her thesis was "Über partielle Differentialgleichungen vierter Ordnung die ein intermediäres Integral besitzen"). It was not until 1971 that the next Norwegian woman would complete a doctorate in mathematics; see Hag & Lindquist (1997).

WS 1903/04	*Calculus II* (4)	94	Miss Fleer; Miss Gamm Miss Paschen Miss Nobbe
	Math. seminar (2), conducted with Karl Schwarzschild: *Selected chapters of hydrodynamics*	22	T. Afanasyeva
SS 1904	*Introduction to the theory of differential equations* (4)	121	Miss Belteneff Miss Kucharskaja[67] Miss Wassiljewa Miss Becker
	Math. seminar (2), conducted with Schwarzschild and Martin Brendel (1862–1939): *Selected chapters of the theory of probability*	35	V. Lebedeva* T. Afanasyeva
WS 1904/05	*Mathematical pedagogy* (4)	115	V. Lebedeva Miss Kucharskaja Miss Belteneff Mrs. Joukovsky Miss Paschen
	Math. seminar (2), with Prandtl. Runge, and Woldemar Voigt (1850–1919): *Theory of elasticity*	23	Miss Kucharskaja
SS 1905	*Elementary mathematics from a higher standpoint: arithmetic, algebra, analysis* (4)	75	
	Math. seminar (2), with Prandtl, Runge, and H. T. Simon (1870–1918): *Electrotechnology*	20	
WS 1905/06	*Projective geometry* (4)	144	Vera Lebedeva Mrs. Joukovsky Miss Haccins Miss Schaeffer Gertrud Lange (b. 1879)[68]
	Math. Seminar (2),	51	V. Lebedeva

[67] Kucharsky (= Kucharskaya), who first appears on Klein's list in 1904, gave her first presentation in the summer semester of 1912. It is in that entry where her first name is mentioned (Xenia). I have not been able to discover any other information about her life.

[68] Gertrud Lange completed her doctorate in physics with the thesis "Beiträge zur Kenntnis der Lichtbogenhysteresis," which was supervised by Hermann Theodor Simon in Göttingen and published in *Annalen der Physik* 337 (1910), pp. 589–647.

	with Hilbert and Hermann Minkowski (1864–1909): *Lectures of Klein on linear differential equation and automorphic functions*		Mrs. Joukovsky Miss Schaeffer
SS 1906	*Theory of functions* (4)	125	Miss Beltenewa Mrs. Joukowsky Miss Potylizyn Miss Schestokow Miss Schirok Miss Stehogolewa Miss Hahn Miss Lehmann Klara Löbenstein
	Math. Seminar (2), with Hilbert and Minkowski: *Differential equations*	32	Miss Beltenewa Mrs. Joukowsky
WS 1906/07	*Theory of functions* (4)	100	A. H. Palmié Anna Johnson (Pell Wheeler) (1883–1966),[69] USA Miss Beltenewa Miss Kucharsky Miss Lehmann
	Math. seminar (2), (Klein, Hilbert, Minkowski, and Herglotz): *Linear differential equations and automorphic functions*	25	A. H. Palmié T. Ehrenfest
SS 1907	*Curves and planes (differential geometry)* (4)	117	V. Lebedeva Olga Polossuchina Miss Gray Miss White Gertrud Lange Margarete Kahn Klara Löbenstein
	Math. seminar (2) (Klein, Hilbert, and Minkowski): *Lectures of Klein on linear differential equations and automorphic functions*	22	
WS 1907/08	*Elementary*	79	Miss Gray

[69] Wheeler, a daughter of Swedish immigrants, is known for her work on linear algebra. She became the chair of the mathematics department at Bryn Mawr in 1925 and was instrumental in bringing Emmy Noether there in 1933. On the close contact between Noether and Wheeler, see Tobies (2003), p. 107.

	mathematics from a higher standpoint: arithmetic, algebra, analysis (4)		Miss E. Meyer Margarete Kahn Klara Löbenstein
	Math. seminar (2), Klein, Carl Runge, Ludwig Prandtl (1875–1953), Emil Wiechert (1861–1928): *Hydrodynamics*	24	
SS 1908	*Elementary mathematics from a higher standpoint: Geometry* (4)	79	Margarete Kahn Klara Löbenstein
	Math. seminar (2), (Klein, Runge, Prandtl, Emil Wiechert): *Hydrodynamics*	15	
WS 1908/09	*Mechanics of point systems* (4)	116	Margarete Kahn Klara Löbenstein Gertrud Lange Iris Runge (1888–1966)[70]
	Math. seminar (2), (Klein, Runge, and Prandtl): *Structural design and mathematics*	14	
SS 1909	*Mechanics of continua*	72	Gertrud Lange Miss I. Lehmann Iris Runge
	Math. seminar (2), (Klein, Runge, and Prandtl): *Elasticity theory and the strength of materials*	8	
WS 1909/10	*Projective geometry* (4)	109	Elisabeth Klein Johanna Droop (b. 1885)[71] Anna Hoffa (b. 1876)[72] Miss Kochler Miss Landsberg Miss I. Weinmeister
	Math. seminar (4), conducted with Felix	9	

[70] See Tobies (2012b).

[71] [BBF] Personalblatt. Droop studied for one semester in Göttingen, passed her examination (math, physics, philosophy) in May of 1912 in Bonn, earned a doctoral degree with a thesis in philosophy there in 1920, and became secondary school teacher in Prussia.

[72] [BBF] Personalblatt. Hoffa completed her teaching examination on February 17, 1911 in Göttingen (philosophy, French, German, and mathematics for the middle grades). In 1930, she became the principal of a girls' school in Frankfurt am Main.

	Bernstein (1878–1965) and Leonard Nelson (1882–1927): *Mathematics and psychology*		
SS 1910	A semester of leave		
WS 1910/11	*Mathematical pedagogy* (2)	67	Clara Dittmar (b. 1880)[73] Käthe Heinemann (b. 1889)[74] Miss M. Kellner Miss B. Meese Miss G. Merker Miss E. Petersen Miss I. Pohlmann Miss Thiele
	Math. seminar (2): *Mathematical pedagogy, especially at elementary schools*	18	Käthe Heinemann* Miss B. Meese* Miss E. Petersen*
SS 1911	*Calculus I* (4)	145	Miss F. Berger Miss E. Bodenburg Miss M. Borchers Hildegard Ehrenberg (b. 1885)[75] Käthe Heinemann Johanna Hilmer (b.1889)[76] Christel Jasmund (b. 1888)[77] Miss K. Klußmann Miss E. Kreuse Berta Kuck (b.1890)[78] Miss W. Lingelbach Miss G. Merker Miss E. Neussel Miss B. Paulssen Clara Pietzsch (b. 1887)[79]

[73] [BBF] Personalbogen. Dittmar passed her teaching examination (geography, mathematics, philosophy) in Göttingen in November of 1912 and became teacher in Wernigerode.

[74] [BBF] Personalbogen. After passing her teaching examination (mathematics, physics, chemistry/mineralogy, botany/zoology), Heinemann earned a doctorate in botany (1922) and became a school principal.

[75] [BBF] Personalbogen. Born in Strasburg, Ehrenberg studied for five semesters in Göttingen, passed her teaching examination (English, mathematics) in November of 1913, and became a teacher in Berlin.

[76] [BBF] Personalbogen. Hilmer became a teacher in Uelzen (mathematics, botany/zoology, physics).

[77] [BBF] Personalbogen. Jasmund completed her teaching examination (mathematics, physics, geography) in Göttingen in July of 1916 and later (as of 1919) worked as a secondary school teacher in Barmen.

[78] [BBF] Personalbogen. After studying in Göttingen, Kuck passed her examination in Münster (physics, mathematics, chemistry/mineralogy) in 1914 and became a secondary school teacher.

[79] [BBF] Personalbogen. Pietzsch studied in Göttingen from 1911 to 1918; her examination took place in February of 1924 (chemistry, physics, mathematics). She taught as a secondary school teacher in Spremberg.

			Miss I. Pohlmann Miss R. Scharlau Irma Schiersand (b. 1883)[80] Miss W. Schönklinz Miss M. Stennes Miss O. Lobanoff
	Math. seminar (2), (Klein and Felix Bernstein): *Insurance mathematics*	8	
WS 1911/12	*Calculus II* (4), from the end of November, held by Wilhelm Behrens (1885–1917) and Hermann Weyl (1885–1955)	145	14 women, including Elisabeth Klein Iris Runge Alma Willers (b. 1881)[81]
	Math. seminar (2), from the end of November, held by Rudolf Schimmack (1881–1912): *History of differential and integral calculus*	31	Iris Runge** Elisabeth Klein* Erna Bockmann* Käthe Heinemann* Miss M. E. Brusstar*
SS 1912	Klein on medical leave		
	Math. seminar (2), announced by Klein (took leave) and Rudolf Schimmack: *IMUK-Literature*		Miss Xenia Kucharsky*[82]

Table 1: Women enrolled in Felix Klein's courses.

The table above does not contain all of the women who studied mathematics at the University of Göttingen at the time. A few women attended only Hilbert's lectures. For instance, we know from the biography of Clara Eliza Smith (1865–1943), who completed her doctorate at Yale in 1904 with the thesis "Representation of an Arbitrary Function by Means of Bessel's Functions," that she conducted post-doctoral research in Göttingen from 1910 to 1911 (while she was a faculty member at Wellesley College). Otherwise, all of Hilbert's female doctoral students mentioned above attended Klein's courses as well.

Although Klein, in 1913, was compelled to retire early for the sake of his health, he continued to hold lectures and seminars until the summer semester of

[80] [BBF] Personalkarte. Schiersand finished her examination (mathematics, geography) in November of 1915 and became secondary school teacher at the Cecilienschule in Breslau.

[81] [BBF] Personalbogen. Willers passed her examination (mathematics, botany/zoology) with distinction in Göttingen in June of 1913 and became a secondary school teacher in Hildenheim, her hometown.

[82] Xenia Kucharsky gave a presentation on July 24, 1912 titled "Über die Organisation der Schulen in Russland" (see *Protokolle* 29, pp. 400–404).

1920. These were mainly devoted to the history of mathematics during the nineteenth century and to questions concerning the theory of relativity. Women known to have attended some of these lectures include Klein's widowed daughter Elisabeth Staiger, her friend Iris Runge,[83] the aforementioned students Erna Bockmann and Käthe Heinemann, M. Jona, Antonie Stern (*1892),[84] Helene Stähelin (*1891) from Switzerland,[85] and others already in possession of a doctoral degree, such as Emmy Noether and the Austrian physicist Gerda Laski (1893–1928).

Noteworthy Trends

It is not often that such thorough records exist about those who attended mathematics courses. On the basis of these lists and other sources, it is possible to draw the following conclusions about the early stages of women studying mathematics in Germany:

(1) The beginnings of women studying mathematics in Germany may be associated with Göttingen, but an international perspective is needed to evaluate the situation. Klein developed an international network of connections that, among other things, brought women to study under his direction. His network was based on his scientific desires to acquaint himself with as many mathematical schools as possible, to maintain the highest international standards for his journal *Mathematische Annalen*, and to commission the foremost international experts to contribute to his monumental encyclopedia of mathematics, the six-volume *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*. He collaborated as a peer-reviewer for several journals of mathematics and he organized international exchanges of scholarship and bibliographical material. Klein himself took numerous research trips abroad and was a member of the most illustrious scientific academies and societies in multiple countries.[86] He served three times as the president of the German Mathematical Society (for three separate yearly terms). When *L'Enseignement mathématique* was founded in 1899 as the first international journal devoted to mathematical instruction, he was made a member of its editorial board. When, in 1908, the first international committee for mathematical instruction was created at the Fourth In-

[83] Iris Runge took part during the winter semester 1914–15 while she was already a secondary school teacher in Göttingen. This is before she would move to Bremen and elsewhere, and before should would become an industrial mathematician in 1923 (see Tobies [2012b]).

[84] A. Stern passed her teaching examination in 1918 (mathematics, physics, chemistry) and completed her doctorate under the supervision of Richard Courant in 1925. Here thesis is entitled "Bemerkungen über asymptodisches Verhalten von Eigenwerten und Eigenfunktionen" ([UAG] Math.-nat. Fak. Prom. S, Vol. I 1922–25, Nr. 35).

[85] Helene Stähelin completed her doctoral thesis – "Die charakteristischen Zahlen analytischer Kurven auf dem Kegel zweiter Ordnung und ihrer Studyschen Bildkurven" – at the university of Basel in 1924.

[86] See [UBG] Cod. MS F. Klein 114 (a collection of Klein's nominations of potential members of academies and scientific societies).

ternational Congress of Mathematicians in Rome, Klein was elected its president (1908–1920), even though he was unable to be present at the meeting.

(2) It was because of his activity on many committees and his many contacts that Klein, for the winter semester of 1893/94, was able to succeed in allowing the first women to attend his courses. Just three years later, he placed the mathematical ability of women on the same level as that of "their male classmates." Klein endorsed the membership of women mathematicians in the German Mathematical Society, and he accepted submissions by women for publication in *Mathematische Annalen*. The first female contributors to this journal were Winston (1895), Scott (1899), Lebedeva (1907, 1909, 1911), Emmy Noether (1915, 1916 [4x], 1917, 1920 [2x], 1921, 1922, 1923, etc.), Tatyana T. Ehrenfest-Afanasyeva (1916), followed by Margarete (Grete) Hermann (1901–1984) in 1926, one of Emmy Noether's doctoral students. Klein supported the initial efforts of women to study in Göttingen with personal letters to the ministry and to the university curator. From 1915 onward, he did much to promote Emmy Noether's career,[87] and he brought aboard Ehrenfest-Afanasyeva as a contributor to his *Encyklopädie*.

(3) Foreign women paved the way for German women. The first female students came from the United States, England, and Russia. They ensured that professors of mathematics could set aside any doubt regarding the intelligence of women, and thus they played a part in the decision of German legislators to allow women to study in an official capacity.

In other countries, secondary education was so developed that women could acquire all of the prerequisites required for university study. They chose to work with Klein in Göttingen because they were encouraged to do so by mathematicians who had been his students, had published in his journal, or had participated in one of his other projects.

Klein had raised his profile in the United States by delivering lectures there during two research trips (1893, 1896). Approximately twelve female North Americans studied under him in Göttingen,[88] one of whom (Winston) earned her doctorate under his supervision, and another (Bosworth) under Hilbert's. American women went on to have careers primarily at women's colleges. A number of them continued to produce valuable research as professors and were themselves able to inspire young women to pursue mathematics as a field of study.

In Russia, advanced courses for women – established, for example, in 1876 in Kazan and in 1878 in St. Petersburg (the so-called Bestuzhev Courses) – provided a solid education (see Borisovna 2003). These courses, too, were taught by mathematicians who had close contact to Klein, among whom D. F. Selivanov (1855–1932) deserves special mention. As of 1895, sixteen well-educated women came to study at the University of Göttingen. Most of them had graduated with a diploma from the Petersburg College for Women, including Helene von Bortkiewicz and her friend Alexandrine von Stebnitzky, who were born to Polish families of officers. Helene von Bortkiewicz was the sister of the statistician and later professor of economics Ladislaus von Bortkiewicz (1868–1931), who had completed his

[87] See especially Tollmien (1990).

[88] The numbers are inexact because not all of the names could be identified.

doctoral degree in Göttingen under the direction of Wilhelm Lexis (1837–1914) and who, by 1895, was already working as a lecturer in Strasburg.

Born in Simbirsk (now Ulyanovsk), Nadeschda von Gernet, who earned her doctorate under Hilbert in 1901 with a dissertation on the calculus of variations,[89] went on to become a lecturer at the aforementioned women's college in St. Petersburg, from which she in turn sent students of her own to Göttingen. An active researcher, she published a book in 1913 on variational calculus and was a member of the German Mathematical Society from 1901 to 1938. After earning her degree, she returned often to Göttingen before the outbreak of the First World War.

Before coming to Göttingen in 1903, Vera Lebedeva had also studied at the women's college in St. Petersburg. Working under Hilbert on the latest field theory of integral equations, she defended her thesis in 1906, and her future husband, the Romanian Alexandru Myller (1879–1965) completed his own in the same year. Both became professors at the University of Iasi in Romania – he in 1910, she in 1918 – and they created an influential school of mathematical thought. With her appointment, in fact, she became the second female professor of mathematics in all of Europe.[90]

Tatyana Afanasyeva, too, had studied at the women's college in St. Petersburg (mathematics and physics). She arrived in Göttingen in 1902 and also met her husband there.

(4) Göttingen became a role model for managing mathematical collaborations, and women were not excluded from this. The couples Chisholm-Young, Lebedeva-Myller, and Afanasyeva-Ehrenfest are examples of scientific couples in which the wife was able to carry on with research after marriage.

The topic of academic couples has been discussed at length by Annette Lykknes and her collaborators.[91] One of the earliest successful examples is the cooperation between Marie and Pierre Curie (1859–1906) in Paris, where Marie Skłodowska (1867–1934) had passed her examinations in physics (1893) and mathematics (1894). Marie Curie became a full professor after the death of her husband and went on to create a famous circle of scientists and thinkers. Although Marie Curie had been awarded two Nobel Prizes, she was never made a member of the Academy of Sciences in Paris.

Regarding the successful couple of Lebedeva and Myller, who both received professorships in mathematics, it should be noted that each of them continued to publish steadily in German and French journals. It should also be pointed out, however, that Lebedeva's Göttingen dissertation received a higher grade than that of her husband, yet he was nevertheless offered a professorship eight years before she was. That said, it is still remarkable that they were able to hold professorships simultaneously at the same university. This was possible in Romania at the time, but not elsewhere.

Grace Chisholm Young and William Henry Young admittedly worked together, but she had to work for him and then had to resign from her position to care for

[89] The evaluations of her dissertation are printed in Tobies (1999).
[90] See Tobies (2004c).
[91] Lykknes, Opitz & van Tiggelen (2012).

their family. Nevertheless, she was pleased to be integrated into the community of mathematicians in Göttingen. In contrast, Saly Ruth Ramler (1894–1993), who was the first Czech woman to hold a doctoral degree in mathematics (earned in Prague in 1919), was quite displeased to be reduced to a housewife and mother. This change in her life took place when she followed her husband, the Dutch mathematician Dirk Struik, to the United States.[92] Tatyana Afanasyeva and Paul Ehrenfest, who had four children as a couple, continued to work and publish together in several places. She was an active teacher of mathematics in Russia and later on in the Netherlands (after the tragic suicide of her husband). The aforementioned Olga Taussky and her Irish husband John Todd (1911–2007) were able to continue their creative mathematical research in the United States. The couple had no children; Olga was five years older, and both ultimately received permanent positions at the Californian Institute of Technology in Pasadena (near Los Angeles).[93]

(5) It is important to stress that Klein was excellent at recognizing the talents of every person in his sphere – women included – and at guiding them toward their own creative achievements. This was the case not only with his female doctoral students but also with other women mathematicians. Annie L. MacKinnon, for example, who during her time in Göttingen (1894–1895) had given five presentations in Klein's seminars, went on to teach mathematics at Wells College in the United States (her courses included spatial geometry, analytic geometry, and differential and integral equations). Encouraged by Klein, she continued to conduct further research; in a letter to him dated January 2, 1897, for instance, she remarked: "As promised, I am writing to you now during the Christmas vacation about the work on number theory that I told you about last summer. I find that I have both the time and desire to undertake such a study and, following your suggestion, I would like to work on it for a year in order to see what I can do with it."[94]

Grace Chisholm Young – who moved to Göttingen as a married woman, became a member of the Göttingen Mathematical Society, and later gave a lecture in one of Klein's seminars (see the table above) – was inspired by Klein to complete a number of works. These include the first English-language book on set theory (written with her husband) and an elementary book on geometry that Klein discussed in his courses, arranged to have translated into German, and recommended in his lectures on elementary mathematics (*Vorlesungen über Elementarmathematik vom höheren Standpunkte*, vol. 2).[95]

During the First World War, Klein presented lectures on the history of mathematics in the nineteenth century. His lectures from the winter semester of 1914–15 and the summer semester of 1915 were recorded by his daughter Elisabeth Staiger, and those from the winter semester of 1915–16 were taken down by Käthe

[92] On developments in the Czech Republic, see Martina Bečvářová's chapter I.3.
[93] See Binder (1998).
[94] [UBG] Cod. MS. F. Klein 10, p. 905. MacKinnon gave a presentation, entitled "Die Smith'sche Curve," in Klein's seminar during the summer semester of 1896.
[95] See http://www.tollmien.com/pdf/chisholm.pdf

Heinemann and Käthe Stähelin for later publication.[96] The editors of Klein's posthumously published *Vorlesungen über die Mathematik im 19. Jahrhundert* (Berlin: Springer, 1926–1927), however, made no mention of the women's contributions. That is a surprise, given that Klein himself always emphasized the work of his associates.

Stimulated by his correspondence with Albert Einstein (1879–1955), Klein initiated a new lecture course on the theory of general relativity in October of 1918, and it was attented by Emmy Noether, Gerda Laski, and other women. Emmy Noether assisted both Klein and David Hilbert with their research on Einstein's theory. Klein recommended Emmy Noether's most significant paper in this field (on invariant variation problems) for publication in the *Nachrichten* of the Academy of Science in Göttingen, and he repeatedly stressed the value of her contribution and results in the edition of his *Collected Papers*.[97]

(6) Another important important issue was that of allowing mathematically educated women to work in appropriate jobs. Felix Klein had high regard for Thekla Freytag (1877–1932), who was the first woman in Prussia to fight for the right to pass the examination for secondary school teachers (for mathematics and scientific subjects), and he wrote about all of the obstacles that she had to overcome to do so in 1905.[98]

Within the framework of the educational reform movement, Klein voiced his opinion on numerous committees, in numerous publications and lectures, and in speeches held as a deputy in the first chamber of the Prussian House of Representatives.[99] Allowed to study as an officially matriculated student, his eldest daughter Elisabeth, mentioned above, reaped the benefits of his efforts. Because she became a widow in 1914, she worked as a secondary school teacher of mathematics, physics, and English, and she ultimately became the principal of a school for girls (though she was demoted when the Nazis came to power in 1933).

With the right of women to matriculate and with the new opportunity of becoming secondary school teachers (and even a principal at a secondary school for girls), the number of female German students of mathematics began to increase. The names of many women who had studied under Klein appear in the Prussian records of female teachers, mentioned above. Because it was long obligatory for female civil servants to remain "celibate," these teachers as a rule remained unmarried or had to leave their positions if they did marry.[100]

With the reform of mathematical and scientific education, which even during his lifetime was known as "Klein's Reform," the number of female students increased in general, for many new career options were made available to women. His new teaching program in applied mathematics also enabled women to become, for instance, an actuary at an insurance company or an industrial mathematician.[101]

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[96] See also Rowe (1992), p. 492.
[97] Klein (1921), pp. 559–560, 565, 584–585.
[98] See Tobies (2017); and Lorey (1909).
[99] See Tobies (1989).
[100] This law applied to women in all official positions; see Deutscher Juristinnenbund (1984), pp. 76–77.
[101] See Tobies & Vogt (2014), and Tobies (2012b).

Klein personally supported Emmy Noether's *Habilitation*, a prerequisite for a professorship at German universities. In a letter to the ministry of education dated January 5, 1919, he wrote that Noether was then the most productive mathematician at the University of Göttingen.[102] In 1919, Noether became the first woman mathematician to achieve this rank. Thus it is clear that Klein, along with other mathematicians in Göttingen, created conditions that would allow women to attain faculty positions at universities.

Klein also emerges as a role model for promoting women if we look at what some of his former students accomplished on this front. A number his students, that is, became the first mathematicians at their respective institutions to supervise female doctoral students. Examples include Adolf Hurwitz and Heinrich Burkhardt in Zurich, Wilhelm Wirtinger and Philipp Furtwängler (1869–1940) in Vienna, Georg Pick (1859–1942) in Prague, Virgil Snyder (1869–1950) at Cornell University in the United States, and Max Winkelmann (1879–1946) at the University of Jena.[103]

(7) Over time, the number of foreign women who earned a doctoral degree in mathematics at German universities fell as the number of German women to do so rose. Up to 1906, seven foreign women (four Russians, two Americans, and an Englishwoman) had defended a mathematical dissertation in Germany (all in Göttingen). From 1907 to 1945, only three foreign women did the same, two from Great Britain (at the Universities of Marburg and Göttingen) and one from Denmark (at the University of Freiburg).[104] The cause of this regression was above all the First World War. Afterwards, many nations, most notably the United States and Russia, established new research centers for mathematics, so that women were then more likely to pursue doctoral research in their home country. Nevertheless, Göttingen remained an important international center for research up until 1933, to which point it continued to attract both male and female mathematicians from abroad.

Translated by Valentine A. Pakis

[102] The letter was published for the first time in Tobies (1991/1992), p. 172.
[103] On George Pick, see Martina Bečvářová's chapter I.3. Regarding Winkelmann, see Tobies & Vogt (2014), p. 195; and Bischof (2014).
[104] See Tobies (2006).

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